

# Implications of Fast Radio Burst Pulse Widths

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## ABSTRACT

The pulse widths, dispersion measures and dispersion indices of Fast Radio Bursts (FRB) impose coupled constraints that all models must satisfy. We show that if the dispersion measures resulted from propagation through the intergalactic medium from cosmological distances and the pulse widths were a consequence of scattering by single thin screens, then the screens' electron densities were  $\gtrsim 20/\text{cm}^3$ ,  $10^8$  times the mean intergalactic density. This problem is resolved if the radiation scattered close to its source, where high densities are plausible. Observation of dispersion indices close to their low density limit of  $-2$  sets a model-independent upper bound on the electron density and a lower bound on the size of the dispersive plasma cloud, excluding terrestrial or Solar System origin. The scattering and much of the dispersion may be attributed to regions about 1 AU from the sources, with electron densities  $\sim 3 \times 10^8 \text{ cm}^{-3}$ . The inferred parameters are only marginally consistent; re-examination of the assumed relation between dispersion measure and distance is warranted. Origin in an ionized starburst or protogalaxy is suggested, but statistical arguments exclude compact young SNR in the Galactic neighborhood. An appendix applies these arguments to PSR J1745-2900 at the Galactic Center. We suggest that its pulse width and angular broadening may be reconciled if we are near a caustic or focal point produced by refraction, rather than by the classic thin sheet scattering model.

*Subject headings:* radio continuum: general — intergalactic medium — plasmas — scattering

## 1. Introduction

Thornton, *et al.* (2013) discovered four fast radio bursts (FRB) whose large dispersion measures (DM) and high Galactic latitudes indicated that their sources were at cosmological distances. FRB 110220 had an observed dedispersed width  $W = 5.6 \pm 0.1 \text{ ms}$  (at a frequency  $\nu = 1300 \text{ MHz}$ ), while only upper limits on  $W$  were found for the remaining three FRB. Burke-Spolaor & Bannister (2014) discovered FRB 011025 for which  $W = 9.4 \pm 0.2 \text{ ms}$ . Fitting  $W \propto \nu^\beta$ , both these FRB had

scattering indices  $\beta$  in agreement with the predicted  $\beta = -4$  for multipath propagation spreading in a refractively scattering plasma medium. Two other FRB, 010621 (Keane, *et al.* 2012) and 121102 (Spitler, *et al.* 2014b), had measured widths but these widths were not attributed to scattering; these FRB occurred at low Galactic latitudes, hinting that they may be Galactic. We do not discuss them explicitly, but their parameters are similar to those of FRB 110220 and FRB 011025, with similar implications if the same assumptions are made.

This paper explores the implications of the assumptions that the dedispersed pulse widths of FRB are a consequence of scattering in intergalactic plasma and that the dispersion measures indicate cosmological distances. Any explanation of these observations must account for two facts: (1) All FRB have dispersion measures within a range of a factor of about two (three if the Lorimer burst (Lorimer, *et al.* 2007) is accepted as an FRB), implying a universal property, not an unusual circumstance such as a line of sight that happens to intersect a rare dense cloud; (2) The dispersion index is very close to  $-2$ , and consistent with exactly  $-2$ , implying an upper bound on the density of the dispersing plasma and a lower bound on its size. These facts are readily accounted for if dispersion occurs in the intergalactic medium, but this appears inconsistent with the pulse broadening, interpreted as the result of scattering.

Section 2 presents our central result, that the scattering responsible for the pulse widths of these FRB did not occur in the general intergalactic medium. Section 3 discusses where the scattering may have occurred. Section 4 obtains limits on the plasma density in the scattering region that can be inferred from the observed dispersion indices. Section 5 sets bounds on the parameters of the scattering region. Section A applies these arguments to Galactic PSR, including the heavily broadened and dispersed PSR J1945–2900 at the Galactic Center. Section 6 considers the implications for models of FRB. Section 7 contains a concluding discussion. Because pulse widths have been measured for FRB110220 and FRB011025, we present numerical results as ordered pairs (110220, 011025).

## 2. Pulse Widths

We make the approximation that FRB (110220, 011025) were at distances  $D = (2.8, 2.2)$  Gpc (Thornton, *et al.* 2013; Burke-Spolaor & Bannister 2014) in a flat static universe. For the estimated redshifts  $z = (0.81, 0.61)$  this only introduces an error of a factor  $\mathcal{O}(1)$ , less than other uncertainties. Following the classic theory of Williamson (1972), we approximate the propagation paths as produced by a single scattering at a distance  $aD$  from us and  $(1 - a)D$  from the source. If the scattering angle  $\Delta\theta \ll 1$  then the angles  $\phi \approx (1 - a)\Delta\theta$  and  $\chi \approx a\Delta\theta$ ; the geometry is shown in Fig. 1.

We assume that the origin of the pulse width  $W$  is dispersion in propagation path lengths. The total propagation delay corresponding to the reported (after subtracting estimated Galactic

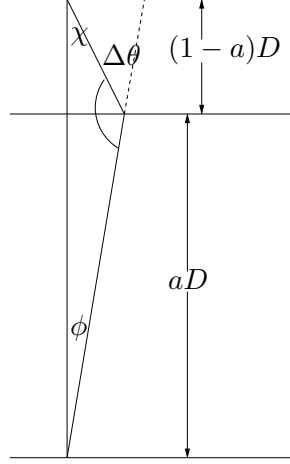


Fig. 1.— Path of scattered radiation

contributions)  $\text{DM} \approx (910, 680) \text{ pc-cm}^{-3}$  (Thornton, *et al.* 2013; Burke-Spolaor & Bannister 2014)

$$\Delta t_{DM} = \frac{2\pi e^2}{m_e c \omega^2} \text{DM} \approx (2.2, 1.7) \text{ s} \approx (400, 180) W \quad (1)$$

at  $\nu = 1300 \text{ MHz}$ , where the last approximate equality compares the  $\Delta t_{DM}$  calculated from the empirical DM to the empirical  $W$ . This assumption implies an assumption about the homogeneity of the intervening medium, in which most of the dispersion is presumed to originate, on scales  $\mathcal{O}(\Delta\theta D) \ll D$  of the separation, perpendicular to the propagation direction, of the weakly scattered paths.

The incremental delay attributable to scattering by an angle  $\Delta\theta$  (Williamson (1972); Kulkarni, *et al.* (2014)) is

$$W \approx \frac{D}{2c} (\Delta\theta)^2 a(1-a). \quad (2)$$

Then

$$\Delta\theta \gtrsim \sqrt{\frac{8cW}{D}} \approx (4 \times 10^{-10}, 6 \times 10^{-10}), \quad (3)$$

the minimum value obtained for  $a = 1/2$ . The angular width of the received radiation

$$\phi \approx (1-a)\Delta\theta = \sqrt{\frac{2cW}{D} \left( \frac{1-a}{a} \right)}. \quad (4)$$

Refraction by a surface whose normal is tilted from the direction of propagation by an angle  $\theta$ , with a ratio  $n$  of refractive indices between the two sides of the surface, leads to a deflection, unless  $|\pi/2 - \theta| \lesssim \mathcal{O}(\sqrt{|1-n|}) \ll 1$ ,

$$\Delta\theta \approx |1-n| \tan \theta. \quad (5)$$

In general  $\tan \theta = \mathcal{O}(1)$ . Taking this as an approximate equality and writing  $2\pi\nu \equiv \omega \gg \omega_p$

$$4 \times 10^{-10} \lesssim \Delta\theta \approx 1 - n \approx \frac{1}{2} \frac{\delta\omega_p^2}{\omega^2}. \quad (6)$$

From the expression for the plasma frequency

$$\delta\omega_p^2 = \frac{4\pi\delta n_e e^2}{m_e}, \quad (7)$$

where  $\delta n_e$  is the magnitude of fluctuations in the electron density, we find

$$\delta n_e \approx \frac{m_e \omega^2}{2\pi e^2} \sqrt{\frac{2cW}{a(1-a)D}} \gtrsim n_{e,min} \equiv \frac{m_e \omega^2}{2\pi e^2} \sqrt{\frac{8cW}{D}} \approx (17, 24) \text{ cm}^{-3}. \quad (8)$$

An elementary calculation shows that (8) also applies to the refractive (dispersion in group velocity) delay if the path is heterogeneous only on a scale  $\sim D$ , while to explain  $W$  as the result of refractive delay in a thin sheet would require much larger  $\delta n_e$ . Hence we consider only the bending of radiation, and not the difference between its group velocity and  $c$ .

### 3. Where Scattered?

The inferred  $\delta n_e$  (8) is more than seven orders of magnitude greater than the maximum cosmologically allowable intergalactic  $\langle n_e \rangle \leq 2 \times 10^{-7} (1+z)^3 \text{ cm}^{-3} = \mathcal{O}(10^{-6} \text{ cm}^{-3})$ . A single scattering screen must have  $\delta n_e \gtrsim 10^7 \langle n_e \rangle$ , a density much too great to be confined in intergalactic space.

The pulse width might be explained as the result of  $\mathcal{O}(10^{14})$  independent uncorrelated scatterings, each by a scatterer with  $\delta n_e \sim \langle n_e \rangle$ . The scattering regions must be  $\lesssim 10^{14} \text{ cm}$  in size. There is no evident source of such fine scale structure in the intergalactic medium, and it would be difficult to maintain because at intergalactic densities the particle mean free paths are  $\mathcal{O}(10^{18} T_{eV}^2 \text{ cm})$ , much longer than the putative structure size. It would be smoothed rapidly by free particle flow, both of electrons and of ions. Henceforth we assume  $\mathcal{O}(1)$  scattering between emitter and detection, rather than a large number of independent scatterings.

An additional argument against the hypothesis of intergalactic scattering is that if scattering were distributed through the intergalactic medium, all FRB should be broadened, with  $W \propto D$  (2). This is inconsistent with the upper bounds on  $W$  found (Thornton, *et al.* 2013) for three other FRB. The large inferred  $\delta n_e$  requires that the pulse was scattered in a dense localized region.

This cannot have been general intergalactic space. The proportionality of  $W$  to  $aD$ , the distance of the scattering medium from the observer (2), implies that scattering within the Galaxy produces roughly the same pulse broadening of FRB as of Galactic pulsars (they differ by the factor  $(1-a)$ , which is  $\mathcal{O}(1/2)$  for typical Galactic sources, but almost exactly unity for extragalactic

sources). The giant nanoshots of the Crab pulsar exclude Galactic broadening of more than 0.4 ns at 9.25 GHz (Hankins, *et al.* 2003; Hankins & Eilek 2007), corresponding (with  $\nu^{-4}$  scaling) to 800 ns at 1400 MHz. The nanoshots of PSR B1937+21 exclude Galactic broadening of more than 15 ns at 1.65 GHz (Soglasnov, *et al.* 2004), corresponding to 30 ns at 1400 MHz. These upper bounds are negligible compared to the observed ms broadening of some FRB, and exclude a significant Galactic contribution.

We consider scattering close to the source, writing  $a = 1 - \epsilon$ , with  $\epsilon \ll 1$ . Then (2) becomes

$$\Delta\theta \approx \sqrt{\frac{2cW}{\epsilon D}} \approx (2 \times 10^{-10}, 3 \times 10^{-10}) \epsilon^{-1/2}. \quad (9)$$

Assume single scattering and combine (6), (7) and (9):

$$\epsilon \delta n_e^2 \approx \frac{cW\omega^4 m_e^2}{2\pi^2 D e^4} \approx (70, 150) \text{ cm}^{-6}. \quad (10)$$

This also illustrates the familiar result  $W \propto \omega^\beta$  with  $\beta = -4$  (consistent with pulsar data; Bhat, *et al.* (2004)) independent of any specific model of the distribution of  $\delta n_e$ , provided all the structure occurs on scales  $\gg \lambda/2\pi$  so that geometrical optics applies. Because  $\Delta\theta \lesssim 1$  we can set a lower bound (too small to be of interest) on the distance of the scatterer from the source:

$$\epsilon D \gtrsim 2cW \approx (3 \times 10^8, 6 \times 10^8) \text{ cm}. \quad (11)$$

#### 4. Dispersion Index

The dispersion index  $\alpha$ , defined by the dispersion delay  $\Delta t \propto \nu^\alpha$ , is a strong constraint on the density of the dispersing plasma. For FRB110220  $\alpha = -2.003 \pm 0.006$  (Thornton, *et al.* 2013) while for FRB 011025  $\alpha = -2.00 \pm 0.01$  (Burke-Spolaor & Bannister 2014). Expansion of the dispersion relation for electromagnetic waves in a cold (nonrelativistic) plasma in powers of  $\omega_p^2/\omega^2 \ll 1$  yields (Katz 2014b)

$$\Delta t = \int \frac{dl}{c} \frac{1}{2} \frac{\omega_p^2}{\omega^2} \left( 1 + \frac{3}{4} \frac{\omega_p^2}{\omega^2} + \dots \right). \quad (12)$$

Then

$$\alpha \equiv \frac{d \ln \Delta t}{d \ln \omega} = -2 - \frac{3}{2} \frac{\omega_p^2}{\omega^2} + \dots = -2 - \frac{6\pi n_e e^2}{m_e \omega^2} + \dots. \quad (13)$$

In order to constrain  $n_e$  in the scattering region we must allow for the fact that it contributes only  $\text{DM}_{\text{scatt}}$  to the (extra-Galactic) dispersion of the pulse. The remainder, perhaps nearly all, is attributed to intergalactic propagation, for which the higher terms in (13) are negligible. From the observed bounds on  $\alpha$ , (13) yields

$$\frac{\omega_p^2}{\omega^2} \frac{\text{DM}_{\text{scatt}}}{\text{DM}} \leq \frac{2}{3} \max(-\alpha - 2) = (0.006, 0.007), \quad (14)$$

where  $\max(-\alpha - 2) \approx 0.01$  is the observed upper bound on  $-\alpha - 2$  for the FRB for which values are reported.

Using (7), (10), (13) and (14),

$$n_e \approx \sqrt{\frac{(-\alpha - 2)cW\omega^6 m_e^3}{12\pi^3 e^6 \text{DM}}} \lesssim (1.6 \times 10^8, 2.6 \times 10^8) \text{ cm}^{-3}, \quad (15)$$

where the inequality results from the most negative values of  $\alpha$  (-2.009, -2.01) permitted by the data.

The lower bound (8) on  $\delta n_e$  may be combined with the upper bound on  $n_e$  implied by the maximum value of  $(-\alpha - 2)$  in (15), assuming  $\delta n_e \sim n_e$ , to yield a lower bound

$$D \gtrsim \frac{24\pi e^2 \text{DM}}{\max(-\alpha - 2)\omega^2 m_e} \sim 10^{14} \text{ cm}; \quad (16)$$

note that the scattering width  $W$  drops out. This bound is more than the statement that the FRB occur outside the inner Solar System.

The electron density and the size  $R$  of the dispersing region are bounded from the plasma dispersion relation, without any consideration of the scattering width:

$$R > \frac{6\pi \text{DM}}{\max(-\alpha - 2)\omega^2 m_e} \sim 2 \times 10^{13} \text{ cm}. \quad (17)$$

This temperature-independent limit applies even to a perfectly homogeneous plasma with no scattering at all, and excludes models that attribute the dispersion to the immediate environment of a star.

## 5. Fluctuation Density and Structure

Make the plausible, but unproven, assumptions  $\delta n_e \sim n_e$  and that the same plasma disperses and scatters the FRB.

### 5.1. Thin Screen

A screen of index  $n$  that refracts radiation may be very thin compared to its distance from the source. Its thickness (assuming transverse structure on the same scale as its thickness) is limited by diffraction to

$$\Delta h \sim \frac{\lambda}{2\pi\Delta\theta} \sim 3 \times 10^{10} \epsilon^{1/2} \text{ cm}. \quad (18)$$

Such a minimal thin screen contributes a negligible amount to DM:

$$\text{DM}_{\min} = \Delta h n_e \sim \frac{c\nu m_e}{e^2} \sim 5 \times 10^{-8} \text{ pc-cm}^{-3}. \quad (19)$$

## 5.2. Thick Plasma

Assume a single scattering but that the electron density implied by the scattering angle  $\Delta\theta$  is present throughout the thickness  $\epsilon D$ , even though the minimal  $\Delta h$  (18) may be orders of magnitude smaller than  $\epsilon D$ . Such a plasma could be an outflowing wind from a point near the FRB source or from the FRB’s progenitor, provided (if from the progenitor) it is asymmetric so that the propagation path is not parallel to its density gradient ( $\tan\theta = \mathcal{O}(1)$  in Eq. 5). Then

$$n_e \epsilon D \equiv \text{DM}_{\text{local}} \leq \text{DM} = (910, 680) \text{ pc-cm}^{-3}, \quad (20)$$

where  $\text{DM}_{\text{local}}$  is the dispersion attributable to matter local to the source that also causes the scattering. Use (10) to eliminate  $n_e$ , obtaining

$$\epsilon D \lesssim \frac{2\pi^2 \text{DM}^2 e^4}{cW m_e^2 \omega^4} \approx (1.3 \times 10^{13}, 4.4 \times 10^{12}) \text{ cm}; \quad (21)$$

Combined with (16) this indicates that, whatever the distance to the FRB, scattering occurs over a small fraction of that distance. the corresponding density of the scattering matter

$$n_e \gtrsim \delta n_e \gtrsim \frac{cW m_e^2 \omega^4}{2\pi^2 \text{DM}_{\text{local}} e^4} \approx (2 \times 10^8, 5 \times 10^8) \text{ cm}^{-3}. \quad (22)$$

The origin of these bounds on the dimensions and density of the plasma are the large value of  $W$ , the assumptions of single scattering and of the identity of the scattering and dispersing plasma. The two bounds (15) and (22) are slightly inconsistent for both FRB, but because of the necessarily rough approximations made, this discrepancy is not significant. Their nearness does indicate that  $n_e$  is near the upper limit of the range allowed by (22) and that a significant fraction of the dispersion measure may be local to the source.

These limits correspond to  $\text{DM}_{\text{local}} \approx \text{DM}$ , in which case DM cannot be used to infer the distance because an unknown fraction, perhaps nearly all, of the dispersion is local to the source. For FRB for which only upper bounds on  $W$  exist, there is neither a lower bound on  $n_e$  nor an upper bound on  $\epsilon D$ . If the scattering plasma contributes only a fraction of the dispersion, then DM should be taken only as that fraction of the total, further tightening the bounds.

If we take the lower bound (11) on  $\epsilon D$  rather than the upper bound (21) then, using (10),

$$\delta n_e \lesssim \omega^2 m_e / (2\pi e^2) \approx 4 \times 10^{10} \text{ cm}^{-3}, \quad (23)$$

independent of the parameters of any particular FRB. This amounts, aside from a factor of two, to the condition that the radiation propagate through the scattering plasma. If  $n_e$  approaches this bound then  $\omega_p \approx \omega$  and  $\Delta\theta = \mathcal{O}(1)$ . Such a dense cloud may also have been the source of the FRB emission (Katz 2014a), but cannot be a major contributor to the dispersion measure because of the arguments made in § 4.

The limits (11) and (22) imply

$$\text{DM}_{\text{local}} \gtrsim \frac{c^2 W^2 \omega^4 m_e^2}{\pi^2 \text{DM} e^4} \approx (0.02, 0.09) \text{ pc-cm}^{-3}, \quad (24)$$

consistent with the lower bound (14) on  $\text{DM}_{\text{scatt}}$ . The two limits (20) and (24) bound the possible range of  $\text{DM}_{\text{local}}$ , and correspond to the bounds (22) and (23) on  $\delta n_e$ . The local contribution to the dispersion measure may, but need not, be very small.

## 6. Constraints on FRB Models

The results of this paper impose a number of constraints on the astronomical environments in which FRB are produced:

### 6.1. Number of FRB Sources

There are two constraints on the number of presently active detectable FRB sources  $N_{\text{sources}} \equiv BT$ , where  $B$  is their birth rate within the volume from which FRB may be detected and  $T$  is their active lifetime (consistent with the known properties of FRB, such as their dispersion measures). If the bursts occur stochastically, without any latency period following a burst, then the absence of coincidences among  $N_{\text{FRB}}$  observed FRB implies

$$N_{\text{sources}} \gtrsim N_{\text{FRB}}^2. \quad (25)$$

The absence of repetitions of any individual FRB implies

$$N_{\text{sources}} \gtrsim \Omega_{\text{FRB}} \tau_{\text{min}} \sim 10^4, \quad (26)$$

where  $\Omega_{\text{FRB}}$  is the all-sky FRB rate and  $\tau_{\text{min}}$  is the empirical lower bound on the repetition time of an individual source. Thornton, *et al.* (2013) estimate  $\Omega_{\text{FRB}} \sim 0.3/\text{s}$  while Kulkarni, *et al.* (2014) estimate  $\Omega_{\text{FRB}} \sim 0.1/\text{s}$ ; the spread between these two values is an indication of their uncertainty.

If the bursts are stochastic then  $\tau_{\text{min}} \sim \tau_{\text{tot}}$ , the total time beams pointed in the *known* directions to FRB, summed over all FRB, without observing a repetition<sup>1</sup>. Law, *et al.* (2014) found no recurrences in  $1.1 \times 10^5$  s of observations of a single FRB, implying a 95% confidence bound  $\tau_{\text{min}} > 2.7 \times 10^4$  s, giving the numerical estimate in (26). On the other hand, if there is a latency period between FRB from a single source then, depending on how observing time was

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<sup>1</sup>It is not necessary that a beam be pointed to a single FRB for this time because, if they all have the same properties, staring in any direction in which an FRB has been observed is equivalent. It is also assumed that localization is good enough that the chance of misidentifying a new source as a repetition of a previously observed source is negligible; for  $15'$  localization and  $N_{\text{FRB}} \sim 10$  this chance is  $\sim 10^{-5}$ .



distributed,  $\tau_{min}$  may be as short as  $\tau_{cont}$ , the longest duration of continuous observation of an individual FRB location without a repetition.

The conditions (25) and (26) may be used to test models of  $N_{sources}$  against the empirical parameters  $N_{FRB}$ ,  $\tau_{min}$  and  $\Omega_{FRB}$ , and thereby to constrain models of the sources, of their astronomical environments, and of their distances. If more than one FRB were observed from the same direction then (25), with the right hand side divided by the number of coincidences, would become an approximate equality.

## 6.2. Supernovae, Soft Gamma Repeaters and Their Remnants

The discovery (Keane, *et al.* 2012; Spitler, *et al.* 2014b) of two apparent FRB at low Galactic latitude suggests they may be cosmologically local and associated with our Galaxy. Kulkarni, *et al.* (2014) suggest an association with the giant flares of SGR, with dispersion originating in the surrounding young SNR, and a lower bound on FRB distances of 300 kpc (for an assumed temperature of the dispersive plasma of 8000°K). The local dispersion measure of a source at the center of a spherical cloud of ionized gas of mass  $M$  and radius  $R$  is

$$DM_{local} = 818 \frac{M}{M_{\odot}} \left( \frac{R}{0.1 \text{ pc}} \right)^{-2} f \text{ pc-cm}^{-3}, \quad (27)$$

where  $f = 1$  for a homogeneous sphere and  $f = 1/3$  for a thin shell, implying  $R \sim 0.1 \text{ pc}$  for a SNR, lost stellar envelope, *etc.*, that provides much of the dispersion measure of a FRB. Note, however, that by (21) scattering by such a cloud cannot also explain the observed pulse widths.

The age and lifetime  $T$  of an expanding cloud

$$T \approx \frac{R}{V} \approx 30 \frac{R}{0.1 \text{ pc}} \frac{3000 \text{ km/s}}{V} \text{ y} \approx 30 \sqrt{\frac{f}{DM_{1000}} \frac{M}{M_{\odot}}} \frac{3000 \text{ km/s}}{V} \text{ y}, \quad (28)$$

where  $V$  is the expansion velocity and  $DM_{1000} \equiv DM/(1000 \text{ pc-cm}^{-3})$ . At  $R = 0.1 \text{ pc}$  only  $\sim 10^{-4} n_{ISM} M_{\odot}$  of interstellar material will have been swept up, for an interstellar density of  $n_{ISM}$  atoms/cm<sup>3</sup>, so  $V$  is nearly the initial explosion velocity. If  $V$  is within the range 3000–30000 km/s of SN ejecta then the age of the dispersing cloud  $T \lesssim 30 \text{ y}$ . If FRB are found within such clouds, then if repetitive bursts are observed their dispersion measures will decrease monotonically and smoothly according to (27) with  $R = Vt$ . The hypothesis that the dispersion is produced by very young cosmologically local SNR is contradicted by the absence of SN within the last  $\sim 30 \text{ y}$  at the high Galactic latitudes of most FRB.

The number of SNR with ages  $t < T$  (28) associated with our Galaxy (out to distances  $\sim 1 \text{ Mpc}$ ) is inferred from the SN rate to be  $N_{SNR \ t < T} \lesssim \mathcal{O}(1)$ . The hypothesis that the dispersion measures of FRB result from propagation through such young and nearby SNR is also contradicted by the fact that no repeaters are observed among seven FRB when only  $\lesssim \mathcal{O}(1)$  SNR young enough

to meet this requirement likely exists within 1 Mpc. Further, the all-sky FRB rate  $\Omega_{FRB} \sim 0.1\text{--}0.3/\text{s}$  would imply a repetition time of an individual source  $\tau \sim N_{sources}/\Omega_{FRB} = N_{SNR} t_{<T}/\Omega_{FRB} \sim 3\text{--}10\text{ s}$ . The hypothesis of such rapid repetitions of FRB is excluded empirically (Law, *et al.* 2014).

If FRB are associated with SN, at a rate of order one-to-one (the FRB do not repeat), comparison of the rates of the two classes of events shows that their distances must be cosmological: The SN rate is estimated (Sharon, *et al.* 2007) to be  $\Omega_{SN} \approx 0.098 \times 10^{-12} M_{\odot}^{-1}\text{y}^{-1}$ . Standard cosmological parameters indicate a local baryon density  $\rho_{baryon} = 1.9 \times 10^{-64} M_{\odot} \text{cm}^{-3}$  and a SN rate  $\Omega_{SN} \rho_{baryon} \approx 1.9 \times 10^{-77} \text{cm}^{-3}\text{y}^{-1}$ . Comparison to the all-sky FRB rate  $\Omega_{FRB} \approx 0.1\text{--}0.3/\text{s}$  indicates that SN out to a distance of  $\sim 1\text{ Gpc}$  must contribute. Unless the volumetric FRB rate is much higher than the SN rate, as might be the case if FRB are giant pulsar pulses (excluded by their dispersion measures, unless at cosmological distances), SGR outbursts (Kulkarni, *et al.* 2014), or other phenomena that repeat many times in their sources’ lifetimes, FRB originate at cosmological distances, even if much of their dispersion measures is local to their sources.

If, on the other hand, many FRB are associated with each SN, we can set a lower bound on the distance out to which FRB are observed:

$$D \gtrsim \left( \frac{3N_{sources}}{4\pi\Omega_{SN}\rho_{baryon}T_{FRB}} \right)^{1/3} \sim 10 \text{ Mpc}, \quad (29)$$

where  $T_{FRB}$  is the FRB-active lifetime of the remnant of a SN; the numerical value assumes  $T_{FRB} \sim 3000 \text{ y}$ , the estimated active lifetime of a SGR. The absence of obvious correlation with cosmologically local structure such as the Coma cluster suggests  $D \gtrsim \mathcal{O}(100) \text{ Mpc}$ .

### 6.3. Inverse Bremsstrahlung

If  $\delta n_e \sim n_e$  and the density  $n_e$  is found over a path length  $\Delta h$  then  $\Delta h = \epsilon D$  and (10) imply an inverse bremsstrahlung optical depth  $\tau_{ff} \propto \epsilon D n_e^2$  in the scattering medium, independent of the particular values of  $\epsilon$  and  $\delta n_e$ . Aside from the medium temperature, this depends only on observed quantities:

$$\tau_{ff} \approx \frac{4}{3} \sqrt{\frac{2\pi}{3k_B T}} \frac{n_e^2 \epsilon D e^6}{k_B T c m_e^{3/2} \nu^2} g_{ff} = \frac{8}{3} \sqrt{\frac{2\pi m_e}{3k_B T}} \frac{W \omega^2 e^2}{k_B T} g_{ff} \approx (2.3, 3.9) \left( \frac{10^7 \text{ K}}{T} \right)^{3/2}, \quad (30)$$

where the Gaunt factor  $g_{ff} \approx 11.5$  (Spitzer 1962). In order that  $\tau_{ff} \lesssim 1$  it is necessary that either  $T \gtrsim 10^7 \text{ K}$  or  $\langle \delta n_e^2 \rangle \gg \langle n_e \rangle^2$  (the scattering matter be a thin dense screen) in a much more dilute medium. The first possibility is consistent with a region of high energy density; the second is also possible but would vitiate the assumption  $\delta n_e \sim n_e$ . The condition (18) is consistent with very thin screens, as in some models (Section A.2) must be responsible for the scattering of PSR J1745-2900. Even if  $W$  is not measured,  $\tau_{ff} \lesssim 1$  still imposes a temperature-dependent constraint on the emission measure  $\int n_e^2 d\ell = \int n_e^2 D d\epsilon$  along the path between the source and the observer (Kulkarni, *et al.* 2014).

#### 6.4. Jeans Limit

If the dispersion occurs in a stable *static* plasma cloud, then the Jeans condition that the cloud be stable against gravitational collapse imposes further constraints on its parameters:

$$\sqrt{\frac{GM}{R}} \lesssim c_s = \sqrt{\frac{5k_B T(1+\mu)}{3m_p}}, \quad (31)$$

where  $c_s$  is the sound speed and  $\mu \approx 0.85$  is the number of electrons per baryon. Substituting  $M \approx R^3 m_p n_e / \mu$  and  $DM \approx n_e R$  (attributing the dispersion to the source's plasma cloud, not the intervening line of sight), we find

$$R \lesssim \frac{5(1+\mu)\mu k_B T}{3GDMm_p^2} \approx 5 \times 10^{21} \frac{T_{8000}}{DM_{1000}} \text{ cm} \quad (32)$$

and

$$n_e \sim \frac{DM}{R} \gtrsim 0.6 DM_{1000}^2 T_{8000}^{-1} \text{ cm}^{-3}, \quad (33)$$

where we normalize the temperature  $T_{8000} \equiv T/8000^\circ\text{K}$  (following Kulkarni, *et al.* (2014)) and the dispersion measure  $DM_{1000} \equiv DM/1000 \text{ pc-cm}^{-3}$ , and assume complete ionization and cosmic abundances. The corresponding mass

$$M \lesssim \frac{25k_B T(1+\mu)^2 \mu}{9G^2 m_p^3 DM} \approx 8 \times 10^7 \frac{T_{8000}^2}{DM_{1000}} M_\odot. \quad (34)$$

The hydrodynamic time

$$T_J \sim \frac{R}{c_s} \lesssim \sqrt{\frac{5k_B T(1+\mu)}{3m_p}} \frac{\mu}{Gm_p DM} \approx 10^8 \frac{T_{8000}^{1/2}}{DM_{1000}} \text{ y} \quad (35)$$

has no explicit dependence on the unknown parameters  $n_e$ ,  $R$  and  $M$ .  $T_J$  is long enough to avoid the statistical problems (Section 6.2) posed by attributing the dispersion measures to young Galactic SNR, whose youth implies that only a very few are active with the observed dispersion measures at any time. The dispersive cloud could be more compact and dense than the bounds (32) and (33), perhaps by a large factor.

These bounds are consistent with dense static compact clouds in the Galactic neighborhood while avoiding the rapid expansion and short lifetime implied by attributing them to rapidly expanding young SNR. Much smaller  $R$  and  $M$  and larger  $n_e$  than the bounds are possible. The bounds also admit a protogalaxy or starburst ionized by an initial generation of hot luminous stars, providing the observed dispersion measures. Such sites may be plausible locales for FRB, but give no clues to the origin of the FRB themselves beyond indicating a relation with massive stars and high rates of star formation and death. As argued in Sections 3 and 5, these clouds cannot be the origin of the observed pulse widths, but may contribute a major part of the total dispersion measures.

## 7. Discussion

The central results (8), (10) and (21) of this paper are that the pulses of FRB 110220 and 011025, the two FRB with pulse widths attributed to scattering, scattered in high density regions close to their sources. We also infer, from the closeness of the dispersion indices to their low density value of  $-2$ , that dispersion occurred in a region where the electron density was close to the bound (22) and that a significant part of the dispersion occurred close to the source. The distances inferred from the dispersion measures are then only upper bounds, although the fact that most FRB occurred at high Galactic latitudes implies that they are either extra-Galactic or very close ( $\lesssim 100$  pc).

These results depend on the assumption  $\delta n_e \sim n_e$ . This assumption could be violated in many ways. For example, the pulse width might have been produced by the reverberation of radio emission in a cavity of size  $< cW$  if the walls of the cavity had a plasma density above the critical density  $n_e \approx 2.1 \times 10^{10} \text{ cm}^{-3}$  for 1300 MHz radiation and the interior had a lower, perhaps much lower, density. However, reverberation would be unlikely *a priori* to produce a scattering index  $\beta \approx -4$ . Alternatively, if scattering occurs in a thin comparatively dense sheet  $\delta n_e \gg \langle n_e \rangle_{LOS}$ , where  $\langle n_e \rangle_{LOS} \equiv DM/D$ , and there may be evidence for such sheets in our Galaxy (A.2, Bower, *et al.* (2014)).

This paper began by assuming that the FRB are at the cosmological distances inferred from their dispersion measures, allowing only for the estimated Galactic dispersions (Thornton, *et al.* 2013). As shown in Section 4, this is only marginally consistent with the dispersion indices. The fact that for both FRB (110220 and 011025) whose pulse widths are attributed to scattering the consistency between (15) and (22) is only marginal should be of concern. It is *a priori* surprising that both objects should be found in the same corner of the allowable parameter space, the range of plasma densities allowed by the pulse widths, which hints at a fundamental problem with the model.

This suggests that for some, as yet undiscovered, FRB, either a significant deviation from the low density plasma dispersion index  $\alpha = -2$  will be found, or there will be a frank inconsistency between the observed  $\alpha$  and that inferred from (13) and (15). Such an inconsistency may require reconsideration of the interpretation of the pulse widths as the effects of scattering or of the dispersion measures as indicating cosmological distances, as Burke-Spolaor & Bannister (2014) and Karbelkar (2014) have done on other grounds. If so, the distances are smaller than inferred from the dispersion measures, perhaps by large factors.

If we reject the inference of cosmological distances then various bounds change. The lower bound (8) on  $\delta n_e$  scales  $\propto D^{-1/2}$ , the estimate (10) of  $\epsilon \delta n_e^2$  scales  $\propto D^{-1}$ , but the bounds on density (14), (15) and (22) are independent of  $D$ . At  $D \sim 30 \text{ kpc}$  (8) becomes  $\delta n_e \gtrsim 6 \times 10^3 \text{ cm}^{-3}$ , consistent with a young SNR (Kulkarni, *et al.* 2014). The bounds (15), (16) and (30) exclude origin in local plasmas, such as meteor trails (the dates of FRB reported by Thornton, *et al.* (2013) do not coincide with meteor showers), lightning and electric discharges.

Finally, we note that radar systems use chirped emission, compressed upon reception into narrow pulses, in order to obtain accurate range measurements without requiring excessive peak transmitted powers. The observation of FRB in a single beam at Parkes, in contrast to perytons (Burke-Spolaor, *et al.* 2011), indicates a distance  $\gtrsim 20$  km, outside the first Fresnel zone, consistent with a radar satellite. There is no obvious reason for a radar to have a chirp  $\omega \propto t^{-1/2}$  as observed, nor is there obvious reason not. However, the observed dispersed pulse durations of several tenths of a second would imply, for monostatic radar, target distances of at least half that many light seconds to avoid interference of the transmission with the received scattered radiation. At such distances  $\sim 10^{10}$  cm the return would be undetectably weak. In contrast, bistatic radar can use arbitrarily long pulses. The pulse repetition intervals would have to have been longer than the lengths of time the radars were anywhere in the 13 beams of the Parkes Multibeam Pulsar Survey (about 0.3 s for a radar in low Earth orbit moving perpendicularly to a beam), yet the pulse durations must have been shorter than the time required to cross a single beam. This explanation would also require at least as many radar satellites, each with a different chirp rate, as FRB because each FRB had a different dispersion measure, or satellites whose chirp rates were variable in some non-obvious manner. This combination of requirements makes the hypothesis of interference by an orbital chirped source implausible.

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## A. Application to Galactic PSR

In this appendix we apply the preceding results to some Galactic pulsars for which  $W$  is constrained empirically.

### A.1. Crab PSR and PSR B1937+21

Both these objects show nanoshots from which upper limits can be placed on broadening by scattering in the plasma on the line of sight. The Crab PSR has nanoshots of width  $\leq 0.4$  ns at 9.25 GHz (Hankins & Eilek 2007), implying  $W \lesssim 0.4$  ns. This sets an upper limit to the lower bound (8):  $n_e \gtrsim n_{e,min}$  where  $n_{e,min} \lesssim 260 \text{ cm}^{-3}$ . Because the actual  $n_{e,min}$  cannot be estimated (only a bound on  $W$  exists, not a measured value), it is not possible to infer anything about  $n_e$  from the scattering argument. The dispersion measure does constrain  $n_e$ , but is consistent with, for example, a perfectly homogeneous medium with no scattering at all. A similar argument for PSR B1937+21, with broadening  $\leq 15$  ns at 1.65 GHz (Soglasnov, *et al.* 2004) leads to  $n_{e,min} \lesssim 40 \text{ cm}^{-3}$ .

The actual values of  $n_{e,min}$  may be orders of magnitude less, making them consistent with interstellar plasma densities. This argument can be inverted to predict  $W$  from known properties of the interstellar medium, with the conclusion that  $W$  is several orders of magnitude less than the

present empirical upper limits.

## A.2. PSR J1745-2900

PSR J1745-2900 at the Galactic Center has  $W = 1.3$  s at 1 GHz (fitted to observations at a range of frequencies from 1.2 GHz to 18.95 GHz with a power law with scattering index consistent with -4) and  $DM = 1778$  pc-cm $^{-3}$  (Spitler, *et al.* 2014a). Its line of sight passes within about  $3''$  (0.1 pc) of Sgr A\*. This is statistically unlikely to be coincidental, and suggests a physical association within that distance of the massive black hole.

We consider the hypothesis that much of the extraordinary scattering and dispersion measure of PSR J1745-2900 are associated with the immediate environment of Sgr A\*, so that  $\epsilon D = 0.1$  pc ( $\epsilon \approx 1.2 \times 10^{-5}$ ). This hypothesis is the natural explanation of the fact that its pulse broadening is several orders of magnitude greater than those of other PSR, such as the Crab PSR and PSR B1937+21. If this broadening were the result of scattering in the general interstellar medium, it would be expected to be roughly comparable for all PSR at comparable distances, not differ by orders of magnitude; the location of PSR J1745-2900 at the Galactic Center is extraordinary, but its propagation path through the interstellar medium is not.

Taking  $\epsilon D = 0.1$  pc and using (10), we find  $\delta n_e \approx 1.2 \times 10^7$  cm $^{-3}$ . However,  $\epsilon D \delta n_e \approx 1.2 \times 10^6$  pc-cm $^{-3}$ , nearly 1000 times greater than the actual dispersion measure (some of which must be attributed to the 8.3 kpc path through the interstellar medium). From this we infer that the scattering occurs in a thin screen whose thickness  $\Delta h \lesssim 4 \times 10^{14}$  cm (a fractional thickness  $\Delta h / \epsilon D \lesssim 10^{-3}$ ). Using (13) we predict a dispersion index

$$\alpha = -2 - 0.0015 \left( \frac{1 \text{ GHz}}{\nu} \right)^2 + \dots \quad (\text{A1})$$

The coefficient of the  $\nu^{-2}$  term is uncertain because the geometry is uncertain. Its measurement would be the first demonstration in an astronomical context of the higher terms in (13).

However, the assumption  $\epsilon D \sim 0.1$  pc implies an angular size, (Fig. 1) using (9),  $\phi \approx \epsilon \Delta \theta \approx 1(\nu/1 \text{ GHz})^{-2}$  mas, in contradiction to the measured angular size, extrapolated to 1 GHz, of 900 mas (Bower, *et al.* 2014). We therefore reject the hypothesis of thin screen scattering with  $\epsilon D = 0.1$  pc.

By comparing the pulse broadening and angular size Bower, *et al.* (2014) concluded that the scattering screen is actually  $5.8 \pm 0.3$  kpc from the Galactic Center ( $a = 0.3$ ); the special environment surrounding Sgr A\* contributes little. Using (8), we find  $\delta n_e \approx 9 \times 10^4$  cm $^{-3}$ , an extraordinary electron density for interstellar space. If, rather than a single scattering sheet there are  $N$  such sheets,  $\delta n_e$  is reduced by a factor  $\mathcal{O}(N^{-1/2})$ .

The requirement (30)  $\tau_{ff} \leq 1$  implies a sheet thickness  $\Delta h \lesssim 7 \times 10^{14} T_{eV}^{3/2} (\nu/1 \text{ GHz})^2$  cm (because  $\delta n_e \sim n_e \propto N^{-1/2}$  this result is independent of  $N$ ) and a contribution to the dispersion

measure  $\leq 20 \text{ pc-cm}^{-3}$ . The thinness of this sheet, and the problem that it is found on the line of sight to PSR J1745-2900 at the Galactic Center but nothing like it is found on lines of sight to other pulsars, is disturbing. The characteristic expansion time  $\Delta h/v_{th} \leq 20T_{eV} \text{ y}$ , suggesting that changes in the pulse broadening, angular size, and possibly DM (more accurately measurable, though proportionately smaller) may be observable over a decade, offering the possibility of an independent test.

Each of the hypotheses for the location of a thin scattering screen meets serious objections: The hypothesis that it is within  $\sim 0.1 \text{ pc}$  of the Galactic Center and of the pulsar implies an angular size nearly three orders of magnitude smaller than observed. The hypothesis that it is in the general interstellar medium at the distance (6 kpc from the Galactic Center and 2 kpc from us) that reconciles the pulse broadening and angular width demands an uncomfortably high screen density and fails to explain why this line of sight, far from the unique Galactic Center, is special: lines of sight to other pulsars don't intersect screens with similar parameters, as shown by the observation that their pulses are broadened (at 1 GHz) by  $< 1 \mu\text{s}$ , compared to about 1 s for PSR J1745-2900.

In order to resolve this problem (the pulse broadening of PSR J1745-2900 is too short for its angular size; equivalently, its angular size is too wide for its pulse broadening) we suggest that the radiation is refracted by plasma near its source, and that we are near a caustic or a focal point. This requires rejection of the *ad hoc* thin screen scattering model, in which there is no correlation between the scattering directions of adjacent points on the screen, and scattered radiation is delayed compared to unscattered radiation, or to that scattered by smaller angles. In contrast, all rays converging on a caustic or optical focus have the same travel time from the source because, by Fermat's Principle that travel time is an extremum, convergence is possible only for rays that have the same (extremum) travel time. This argument assumes the travel time is a continuous function of angular displacement (the lens properties must be continuous functions of displacements perpendicular to the rays), and does not apply to Fresnel lenses.

The rays may have an arbitrarily broad angular distribution that depends on the geometry of the refracting medium. Sources for which we are on a caustic or at a focus are brightened, perhaps explaining why the sole detected Galactic Center pulsar satisfies this unusual and unexpected condition. The observation of nonzero scattering widths for PSR J1745-2900 with scattering index  $\beta = -3.8 \pm 0.2$  (Spitler, *et al.* 2014a) indicates that there is a contribution to the detected radiation that is described by the thin sheet scattering model.

These arguments cannot be applied to FRB because no measurements of their angular sizes exist. Interferometric detection of FRB (Law, *et al.* 2014) might permit measurements of their angular sizes, and be a critical test. A more quantitative investigation is the subject of ongoing work, but is beyond the scope of this paper.

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